AD-A047 682 MARYLAND UNIV COLLEGE PARK COMPUTER SCIENCE CENTER F/G 9/4

SHAPE COMPLETION.(U)

AUG 77 W RUTKOWSKI AFOSR-77-3271

UNCLASSIFIED TR-564 NL

1 OF 1

AD
A047682

END

DATE FILMED

1-78

DDC

AFOSR-TR- 77 - 1292

11 B.S.

See 1473 in back

COMPUTER SCIENCE TECHNICAL REPORT SERIES

UNIVERSITY OF MARYLAND
COLLEGE PARK, MARYLAND
20742

Approved for public release;
distribution unlimited.

(11)

TR-564 August 1977
AFOSR-77-3271

SHAPE COMPLETION

Wallace Rutkowski
Computer Science Center
University of Maryland
College Park, MD 20742

ABSTRACT

This report describes some simple techniques for smoothly filling in gaps in object contours. The first technique considered was recently proposed by Ullman [1]; it constructs the completion of the contour using two arcs of circles that are tangent to the gap ends and to each other, and that have minimum total curvature. An analysis of this technique is presented, and examples of its use are given. A second technique uses cubic polynomial completions; when suitably constrained, this technique yields very reasonable completions.

The support of the Directorate of Mathematical and Information Sciences, U. S. Air Force Office of Scientific Research, under Grant AFOSR-77-3271, is gratefully acknowledged, as is the help of Mrs. Shelly Rowe in preparing this paper.

This report describes some simple techniques for filling in gaps in object contours. Such gaps occur in many classes of images as a result of occlusion, shadows, etc. Many levels of knowledge may be broght to bear upon the problem of filling in a missing segment of an object boundary. The techniques considered here utilize only local information. Specifically, we are given two points through which the contour must pass, and the directions in which the contour is going at those points.

1. The Ullman Method

The first technique we will consider was proposed by S. Ullman [1]; the following description is quoted from Ullman's paper:

> Subjective contours are perceived when the visual system fills in the gap between distinct edges. Other examples of filling-in processes are the perceived trajectory in optimal Beta motion and the continuation of the visual field across retinal scotomata. The study of these filling-in processes has concentrated to date on the triggering conditions. For instance, under what conditions will a complete trajectory be perceived between successively presented stimuli, or what conditions will enhance the generation of subjective contours between distinct edges. Another important but hitherto neglected problem posed by the filling-in phenomena concerns the shape of the filled-in contours and trajectories. Since subjective contours are synthesized by the visual system the examination of their shape might provide clues about the nature of the mechanism that generates them. In this paper I shall first examine the shape of subjective contours and then suggest a network capable of generating these shapes.
>
> Of the infinitely many loci passing through two given edges, which is the one chosen by the visual system? Let me start the quest for an answer by stating three observations and an hypothesis which will

serve as guidelines for narrowing down the range of possible shapes to a unique one. The resulting curves will then be compared with actually perceived filled-in contours. The four guidelines are:

1. Isotropy: As far as the filled-in contours are concerned the visual field seems approximately homogeneous and isotropic: The filled-in contours produced by a given figure do not change in shape when the figure is translated or rotated.

2. Smoothness: except for some special cases of filled-in corners, the generated curves are smooth, that is, differentiable at least once.

3. Minimum curvature: This guideline is inspired by the resemblance of the filled-in contours to a thin doubly cantilevered beam or, alternatively, to the curve known in approximation theory as a cubic spline. Both curves are, in some sense, loci of minimum curvature. The shape of a subjective contour passing through two given edges whose orientation difference is much less than $\pi/2$ closely resembles the cubic spline passing through this boundary edges. At this point I only wish to state this resemblance informally: the minimum curvature property will receive a formal treatment in the sequel.

4. The locality hypothesis: The operation by which a boundary edge is extended to generate a subjective contour is assumed to be local in nature. That is, it depends only on the end part of the given edge to be extended and not on the shape of the entire edge.

The four guidelines are sufficient to resolve the shapes of the filled-in contours.

Let the set of possible extensions [of a contour] A be L1, L2,...,Lk. The set of curves L1,...,Lk possesses the following property. If you take any curve Li and cut it at some point P, then the shape of P-Li, the continuation of Li from P onwards, is congruent to one of the curves L1,...,Lk. One can meet this requirement by taking all the Li's to be <u>arcs of some circles</u> (including the limit case of a straight line), since any part of a circle is congruent to any other part of equal length. Under certain conditions, the circles solution is the <u>only</u> set of curves obeying the above requirement; however, I shall not diverge here to examine these conditions.

Since the produced curves must all be <u>tangent</u> to the edge A to meet the smoothness requirement, the prediction is that the filled-in contour is composed of the arcs of two circles, one tangent to one edge, the other tangent to the second edge. An additional restriction is imposed by the smoothness guideline, which requires that the two arcs share a common tangent at their meeting point. There is still, however, an infinite number of arc pairs satisfying all the above conditions. Following the minimum curvature hypothesis let us pick, out of all the admissible pairs, the pair which minimizes the total curvature. The total curvature of a curve is defined here as the integral $\int (d\alpha/dl)^2$ over the curve, where α is the slope of the

curve. The value of $d\alpha/dl$ at a given point P is known as the local curvature at that point. In the sequel, however, the word "curvature" will refer to total curvature unless otherwise stated.

Ullman goes on to describe a locally connected network which computes these contours. In the absence of such a network, a conventional approach must be employed to compute the contour. We will now describe such an approach.

2. Implementation of the Ullman Method

In Figure 1a we have a circular arc emanating from point P in direction μ_1 and terminating at point Q in direction μ_2. The angle α is measured from the initial direction vector to a vector pointing from the initial point of the arc to the terminal point. Note that the total angular change $(\int_{\mu_1}^{\mu_2} d\theta)$ is 2α. This holds true for $\alpha \in [-\pi, +\pi]$. We run into difficulty if we use a different range for α. For example, in Figure 1b we have $\alpha = -\pi/2$, and the arc goes through an angular change of $2\alpha = -\pi$. However, if we instead measured α as $+ 3\pi/2$, and constructed a circular arc with total angular change 2α, we obtain the situation shown in Figure 1c. The vector from the initial to the terminal point of the arc now points in a direction opposite to the direction defined by the angle $3\pi/2$. Of course, it is obviously foolish to construct completions using circular arcs with angular changes exceeding 2π in magnitude. Therefore we can restrict 2α to $(-2\pi, +2\pi)$, or $\alpha \in (-\pi, +\pi)$, for the arcs used in our contours.

It is a matter of elementary geometry to show that for Figure 1a,

$$\int \left(\frac{d\theta}{d\ell} \right)^2 d\ell = \frac{4\alpha \sin\alpha}{D} .$$

We can now define our problem with the aid of Figure 2a. The contour we wish to compute consists of two circular arcs which join at point J. For angles φ_1, θ_1, and α_1, associated with the point P_1, let us consider counterclockwise rotation as positive, while for φ_2, θ_2, and α_2, we will consider

clockwise rotation positive. Although this may seem confusing at first, it leads to symmetry in the equations developed below. Thus in Figure 2a, Θ_1, φ_1, Θ_2, φ_2 would be positive, and α_1 and α_2 would be negative. All angles are in $[-\pi, +\pi]$.

Let us now consider the total angular change in following the contour from P_1 to P_2. In going from P_1 to point J, the angular change (considering counterclockwise as positive) is $2\alpha_1$, as discussed earlier in connection with Figure 1. As we go from J to P_2, the angular change (again considering counterclockwise as positive) is $2\alpha_2$. At first this may seem puzzling, since α_2 is measured considering clockwise rotation as positive. Indeed, the counterclockwise rotation would be $-2\alpha_2$ if we were proceeding from P_2 to J; however, since we are moving from J to P_2, the counterclockwise rotation is $2\alpha_2$. Thus the total angular change, travelling from P_1 to P_2, is $2\alpha_1 + 2\alpha_2$, where the angular change is measured according to the conventions used at P_1. The total angular change from P_1 to P_2 is also given by $-\varphi_1 - \varphi_2$ (again measured with respect to the conventions at P_1). For example, in Figure 2b the total average change from P_1 to P_2 is $-\pi$. Thus we have $2\alpha_1 + 2\alpha_2 = -(\varphi_1+\varphi_2)$. Of course, we can add or subtract multiples of 2π to the total angular change required to go from P_1 to P_2, so that in general

$$2\alpha_1 + 2\alpha_2 = -(\varphi_1+\varphi_2) + 2n\pi$$

or, defining $\Phi_{AVG} = \dfrac{\varphi_1+\varphi_2}{2}$

$$\alpha_1 + \alpha_2 = -\Phi_{AVG} + n\pi.$$

Therefore, since $\Theta_1 = \varphi_1 + \alpha_1$ and $\Theta_2 = \varphi_2 + \alpha_2$, we have $\Theta_1 + \Theta_2 = \Phi_{AVG} + n\pi$. Since Φ, and $\Phi_2 \varepsilon[-\pi,+\pi]$, clearly $\Phi_{AVG} \in [-\pi, +\pi]$. In fact, Φ_{AVG} can only equal $\pm\pi$ if both Φ_1 and $\Phi_2 = \pm\pi$. In this case, illustrated in Figure 2c, the entire technique breaks down, as there is obviously no pair of circular arcs which will join P_1 and P_2 as required. Hence, $\Phi_{AVG} \in (-\pi, +\pi)$ for any situation in which it is possible to construct the contour. Now if we wish to form a triangle $P_1\ J\ P_2$ as in Figure 1a, then $|\Theta_1 + \Theta_2| \leq \pi$. Thus:

n can equal +1 only if $\Phi_{AVG} \leq 0$

n can equal -1 only if $\Phi_{AVG} \geq 0$

$|n|$ can never exceed 1.

To summarize, Θ_1 and Θ_2 obviously determine the point J at which the circular arcs join, and we have now derived a constraint on Θ_1 and Θ_2. It is a constraint which has a simple geometric interpretation. The locus of points J for which $\Theta_1 + \Theta_2 = \Phi_{AVG} + n\pi$, with n restricted as given above, is a circle. For example, in Figure 3 we show cases where $\Phi_{AVG} = \pi/2$, $\pi/4$, and 0. The last of these cases is unusual in that we can only form degenerate straight line triangles. Although it is possible to compute contours using the constraint $\Theta_1 + \Theta_2 = \Phi_{AVG} + n\pi$ with non-zero n, we can get into serious difficulties when doing so. Note that we are attempting to find a contour which fits the above constraint and has minimum curvature. Now consider the case shown in Figure 4a. If we look at the family of completions defined by

$\Theta_1 + \Theta_2 = \Phi_{AVG} = 0$, we see that they look as shown in Figure 4b, and the minimum curvature contour obeying the constraint $\Theta_1 + \Theta_2 = 0$ is as shown in Figure 4c, as we would expect from symmetry. Now, however, consider the family of contours obeying the constraint $\Theta_1 + \Theta_2 = +\pi$. These contours look as shown in Figure 4d. The curvature of a circular arc is

$\int \left(\frac{d\Theta}{d\ell} \right)^2 d\ell = \frac{2\alpha}{R}$ where 2α is the angular change along the arc, and R is the radius of curvature. Note that in the family of contours illustrated in Figure 4d, α_1 and α_2 remain fixed at $\pi/2$, while the radii can increase without bound. Thus there is no finite solution in the case $\Phi_{AVG} = 0$, or $\varphi_1 = -\varphi_2$. This special symmetric case with $\varphi_1 = \varphi_2$ is not the only problem. Consider the "almost symmetric case" in Figure 5a. The locus of points at which the arcs can join is illustrated in Figure 5b. If we examine the family of contours obeying $\Theta_1 + \Theta_2 = \Phi_{AVG}$, we get a minimum solution which is "almost symmetric" (Figure 5c). However, there exist solutions obeying $\Theta_1 + \Theta_2 = \Phi_{AVG} - \pi$ which have lower curvature than the curve above (assuming Φ_{AVG} close enough to 0), as in Figure 5d. Although in this case $\Phi_{AVG} \neq 0$, so we can find a finite minimum curvature contour obeying $\Theta_1 + \Theta_2 = \Phi_{AVG} = -\pi$, we find that the shape of the contour is not "almost symmetric" as we would intuitively like it to be. In fact, the closer we approach the symmetric case $\varphi_1 = -\varphi_2$, the larger the minimum contour obeying $\Theta_1 + \Theta_2 = \Phi_{AVG} - \pi$ grows.

Because of this, we shall from now on use $\Theta_1 + \Theta_2 = \Phi_{AVG}$. Remembering again that Θ_1 and Θ_2 are angles in the triangle $P_1\ J\ P_2$, it is clear that Θ_1 and Θ_2 must have the same sign or we cannot form the triangle. Therefore, Θ_1 and Θ_2 can only take on values between 0 and Φ_{AVG}. There is one further point to be checked: $\alpha_1 = \Theta_1 - \varphi_1$ and $\alpha_2 = \Theta_2 - \varphi_2$, and we cannot use values of Θ which would put α_1 or α_2 out of the range $(-\pi, +\pi)$. We see that if Θ_i takes on values between 0 and Φ_{AVG}, α_i will take on values between $-\varphi_i$ and $\Phi_{AVG} - \varphi_i$. These are both in $(-\pi, +\pi)$, so we may consider any Θ_1, Θ_2 as between 0 and Φ_{AVG}. (As a further note, if we used $\Theta_1 + \Theta_2 = \Phi_{AVG} \pm\pi$, we would find that some values of Θ_1 and Θ_2 would have to be excluded because they cause α_1 or α_2 to fall out of $(-\pi, +\pi)$.)

We can also observe that although from the point of view of determining the location of point J it does not matter if we use Θ_1, $\Theta_1 + 2\pi$, etc., we must remember that $\alpha_i = \Theta - \varphi_i$, and so adding a multiple of 2π to Θ_i also adds it to α_i. We have already determined that values of Θ_i between 0 and Φ_{AVG} will cause α_i to be in $(-\pi, +\pi)$, so clearly we cannot add multiples of 2π to these values of Θ_i without putting the α_i out of $(-\pi, +\pi)$.

Finally, remembering the observation on curvature made in connection with Figure 1, we find that the total curvature in Figure 2a is

$$\frac{4\alpha_1 \sin\alpha_1}{D_1} + \frac{4\alpha_2 \sin\alpha_2}{D_2}$$

But, using the law of sines,

$$D_1 = D\sin\Theta_2/\sin(\Theta_1+\Theta_2) \text{ and}$$

$$D_2 = D\sin\Theta_1/\sin(\Theta_1+\Theta_2) \text{ so we have}$$

$$\frac{4\sin(\Theta_1+\Theta_2)}{D}\left(\frac{\alpha_1\sin\alpha_1}{\sin\Theta_2} + \frac{\alpha_2\sin\alpha_2}{\sin\Theta_1}\right)$$

Since $\Theta_1 + \Theta_2 = \Phi_{AVG}$, and $\alpha_i = \Theta_i - \varphi_i$, we can now state our problem analytically as finding the minimum value of

$$\frac{4\sin\Phi_{AVG}}{D}\left(\frac{[\Theta_1-\varphi_1]\sin[\Theta_1-\varphi_1]}{\sin\Theta_2} + \frac{[\Theta_2-\varphi_2]\sin[\Theta_2-\varphi_2]}{\sin\Theta_1}\right)$$

subset to the constraint $\Theta_1 + \Theta_2 = \Phi_{AVG}$, where Θ_1 and Θ_2 can only take on values between Θ and Φ_{AVG}. Clearly, Φ_{AVG} is a special case, as this implies $\Theta_1 = \Theta_2 - 0$. We will treat this case later.

If we define $\Phi_{DIF} = \frac{\varphi_1-\varphi_2}{2}$ then we can write

$$\Theta_1 - \varphi_1 = \Phi_{AVG} - \Theta_2 - \varphi_1 = \frac{\varphi_1}{2} + \frac{\varphi_2}{2} - \Theta_2 - \varphi_1$$

$$= \frac{\varphi_2}{2} - \frac{\varphi_1}{2} - \Theta_2 = -\Phi_{DIF} - \Theta_2$$

and $\Theta_2 - \varphi_2 = \Phi_{DIF} - \Theta_1$. Hence,

$$\frac{(\Theta_1-\varphi_1)\sin(\Theta_1-\varphi_1)}{\sin\Theta_2} + \frac{(\Theta_2-\varphi_2)\sin(\Theta_2-\varphi_2)}{\sin\Theta_1}$$

$$= \frac{(-\Phi_{DIF}-\Theta_2)\sin(-\Phi_{DIF}-\Theta_2)}{\sin\Theta_2} + \frac{(\Phi_{DIF}-\Theta_1)\sin(\Phi_{DIF}-\Theta_1)}{\sin\Theta_1}$$

$$= (-\Phi_{DIF}-\Theta_2)(\sin(-\Phi_{DIF})\cos(-\Theta_2) + \cos(-\Phi_{DIF})\sin(-\Theta_2))/\sin\Theta_2$$

$$+ (\Phi_{DIF}-\Theta_1)(\sin(\Phi_{DIF})\cos(-\Theta_1) + \cos(\Phi_{DIF})\sin(-\Theta_1))/\sin\Theta_1$$

$$= (-\Phi_{DIF}-\Theta_2)(-\sin\Phi_{DIF}\cot\Theta_2-\cos\Phi_{DIF})$$

$$+ (\Phi_{DIF}-\Theta_1)(\sin\Phi_{DIF}\cot\Theta_1-\cos\Phi_{DIF})$$

$$= \sin\Phi_{DIF}[(\Phi_{DIF}+\Theta_2)\cot\Theta_2 + (\Phi_{DIF}-\Theta_1)\cot\Theta_1] + \Phi_{AVG}\cos\Phi_{DIF} .$$

The total curvature can thus be written as $\frac{4\sin\Phi_{AVG}}{D}$ X the above expression. If, for the moment, we ignore the special cases in which Φ_{AVG} and/or Φ_{DIFF} are zero, we see that this function goes to infinity as either Θ_1 or Θ_2 approach zero. Since the minimum curvature obviously does not occur at either end of the range in which Θ_1 and Θ_2 must lie, we must look for a local minimum. The derivative of the above expression is

$$\frac{4\sin\Phi_{AVG}\sin\Phi_{DIF}}{D}\frac{d}{d\Theta_1}[(\Phi_{DIF}+\Theta_2)\cot\Theta_2 + (\Phi_{DIF}-\Theta_1)\cot\Theta_1] .$$

Since we are assuming that Φ_{AVG} and Φ_{DIF} are nonzero, $4\sin\Phi_{AVG}\sin\Phi_{DIF}/D$ is nonzero, and we must have

$$\frac{d}{d\Theta_1}[(\Phi_{DIF}+\Theta_2)\cot\Theta_2 + (\Phi_{DIF}-\Theta_1)\mathrm{cost}\Theta_1] = 0 .$$

Since $\Theta_1 + \Theta_2 = \Phi_{AVG}$, $\frac{d\Theta_2}{d\Theta_1} = -1$, and we must obtain

$$\frac{\Phi_{DIF}+\Theta_2}{\sin^2\Theta_2} + \frac{-\Phi_{DIF}+\Theta_1}{\sin^2\Theta_1} - \cot\Theta_1 - \cot\Theta_2 = 0 .$$

Multiplying through by $\sin\theta_1 \sin\theta_2$ gives

$$(\Phi_{DIF}+\theta_2)\frac{\sin\theta_1}{\sin\theta_2} + (-\Phi_{DIF}+\theta_1)\frac{\sin\theta_2}{\sin\theta_1} - \cos\theta_1\sin\theta_2 - \cos\theta_2\sin\theta_1 = 0$$

$$(\Phi_{DIF}+\theta_2)\frac{\sin\theta_1}{\sin\theta_2} + (-\Phi_{DIF}+\theta_1)\frac{\sin\theta_2}{\sin\theta_1} - \sin\Phi_{AVG} = 0 .$$

With the derivative in this form, we can see that as $\theta_1 \to 0$, the derivative goes to infinity with the same sign as $-\Phi_{DIF}$, and as $\theta_2 \to 0$, the derivative goes to infinity with the same sign as Φ_{DIF}. (Remember that θ_1 and θ_2 must have the same sign, so $\sin\theta_1/\sin\theta_2$ and $\sin\theta_2/\sin\theta_1 > 0$). Since we know that θ_1 and θ_2 must be between 0 and Φ_{AVG}, and we know the behavior of the derivative at these points, it is easy to solve the above equation numerically by bisection.

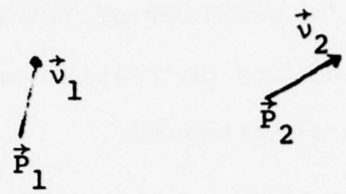
We still have two special cases to take care of. If $\Phi_{DIF} = 0$, then $\varphi_1 = \varphi_2$. In this case, the contour is just a single circular arc. This can, of course, be viewed as two circular arcs of equal radius joined at any point we choose along the contour. Note that the derivative is zero, as expected, since the contour and curvature remain the same regardless of where we choose to join the arcs. If Φ_{DIF} is nonzero, we must still have $\Phi_{AVG} = 0$. In this case $\varphi_1 = -\varphi_2$. Since θ_1 and θ_2 must lie between 0 and φ_{AVG}, $\theta_1 = \theta_2 = 0$, so the arcs must join at some point on a line between P_1 and P_2 (Figure 6). The curvature is

$$\frac{\varphi_1 \sin\varphi_1}{D_1} + \frac{-\varphi_1 \sin -\varphi_1}{D_2} = \varphi_1 \sin\varphi_1 \left(\frac{1}{D_1} + \frac{1}{D_2}\right) \text{ where } D_1 + D_2 = D, \text{ and}$$

D_1 and D_2 must be between 0 and D. It is not difficult to show that the minimum value occurs at $D_1 = D_2 = D/2$. (Note that since $\Phi_{DIF} \neq 0$, $\varphi_1 \sin\varphi_1 \neq 0$.) We thus obtain the result we would expect due to symmetry. We could also arrive at the last result by examining the behavior of the expressions we derived for the curvature and its derivative as $\varphi_1 \rightarrow -\varphi_2$, although this approach is more complicated.

3. Cubic Interpolation

We can take another approach, using polynomials. Suppose we have two points $\vec{P}_1$ and $\vec{P}_2$, and two vectors $\vec{\nu}_1$ and $\vec{\nu}_2$, as in the diagram below.



We can define a curve $(x(t),\ y(t))$ which passes through point $\vec{P}_1$ with vector derivative $\vec{\nu}_1$, and passes through point $\vec{P}_2$ with vector derivative $\vec{\nu}_2$ in the following way:

Let $\vec{P}_1 = \begin{pmatrix} P_{1x} \\ P_{1y} \end{pmatrix}$, $\vec{P}_2 = \begin{pmatrix} P_{2x} \\ P_{2y} \end{pmatrix}$, $\vec{\nu}_1 = \begin{pmatrix} \nu_{1x} \\ \nu_{1y} \end{pmatrix}$, $\vec{\nu}_2 = \begin{pmatrix} \nu_{2x} \\ \nu_{2y} \end{pmatrix}$

and let parameter $t \in [0,1]$.

Then we want

$$\begin{pmatrix} x(0) \\ y(0) \end{pmatrix} = \vec{P}_1,\ \begin{pmatrix} x'(0) \\ y'(0) \end{pmatrix} = \vec{\nu}_1,\ \begin{pmatrix} x(1) \\ y(1) \end{pmatrix} = \vec{P}_2,\ \begin{pmatrix} x'(1) \\ y'(1) \end{pmatrix} = \vec{\nu}_2$$

We can satisfy the above conditions by defining

$$x(t) = A_x t^3 + B_x t^2 + C_x t + D_x$$

$$y(t) = A_y t^3 + B_y t^2 + C_y t + D_y$$

where

$$A_x = 2P_{1x} - 2P_{2x} + \nu_{1x} + \nu_{2y}$$

$$B_x = 3P_{2x} - 3P_{1x} - 2\nu_{1x} - \nu_{2x}$$

$$C_x = \nu_{1x}$$

$$D_x = P_{1x}$$

$$A_y = 2P_{1y} - 2P_{2y} + \nu_{1y} + \nu_{2y}$$

$$B_y = 3P_{2y} - 3P_{1y} - 2\nu_{1y} - \nu_{2y}$$

$$C_y = \nu_{1y}$$

$$D_y = P_{1y}$$

Of course, since we are really only interested in matching the directions of $\vec{\nu}_1$ and $\vec{\nu}_2$ at $\vec{P}_1$ and $\vec{P}_2$, the magnitudes of $\vec{\nu}_1$ and $\vec{\nu}_2$ are free parameters.

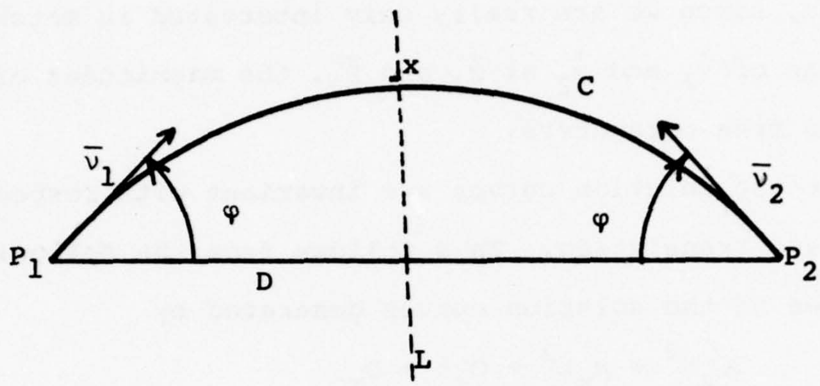
Note that the solution curves are invariant with respect to rotation and translation. This follows from the following two properties of the solution curves generated by

$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} A_x t^3 + B_x t^2 + C_x t + D_x \\ A_y t^3 + B_y t^2 + C_y t + D_y \end{pmatrix}:$$

1) This class of curves is obviously closed under rotation and translation.
2) Any pair of points and vectors, $\vec{P}_1$, $\vec{P}_2$, $\vec{\nu}_1$, $\vec{\nu}_2$ generates a unique solution curve.

In other words, suppose we have points P_1, P_2 and vectors ν_1, ν_2, and the corresponding solution curve is $(x(t), y(t))$. Then if we apply a rotation and translation to P_1, P_2, ν_1, ν_2 to get P_1', P_2', ν_1', ν_2', clearly the curve $(x(t), y(t))$ taken through the same rotation and translation will yield a solution curve to the problem with P_1', P_2', ν_1', ν_2'. But since P_1', P_2', ν_1', ν_2' determine a unique solution curve $(x'(t), y'(t))$, this curve must be a rotated and translated version of $(x(t), y(t))$.

In order to implement this method, it was of course necessary to devise a means of determining the two free parameters (the magnitudes of the vectors $\vec{\nu}_1$ and $\vec{\nu}_2$). The method used to determine these parameters is based on the following observation. Consider the figure below:



L is the perpendicular bisector of line segment $\overline{P_1P_2}$, and C is a circular arc. It is easily shown that a polynomial based completion, generated by the technique described previously, will pass through point x (the same point through which a circular completion passes) when

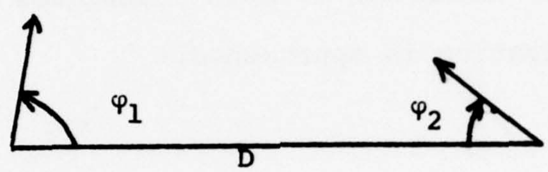
$$|\vec{\nu}_1| = |\vec{\nu}_2| = \frac{2D}{1+\cos\Theta}$$

The resulting polynomial completion closely approximates a circular completion until φ becomes much greater than $\pi/2$. Therefore, to facilitate a comparison between the two methods of generating completions, it was decided to determine the magnitudes of $\vec{\nu}_1$ and $\vec{\nu}_2$ in such a way that in the symmetrical case diagrammed above $|\vec{\nu}_1| = |\vec{\nu}_2| = 2D/(1+\cos\varphi)$. There are obviously many ways to do this. The one used in the program

is

$$|\vec{v}_1| = |\vec{v}_2| = \frac{2D}{1 + \left(\frac{\cos\varphi_1 + \cos\varphi_2}{2}\right)}$$

(see the figure below).



4. Examples

To generate examples on which to run the programs, we let both φ_1 and φ_2 take on values in multiples of $\pi/4$. This, of course, results in eight values for both φ_1 and φ_2, and yields 64 cases. Many of these cases are, however, reflections of other cases. Therefore only a set of cases which are unique up to reflection are shown. (Of course, the case $\varphi_1 = \pi, \varphi_2 = \pi$ is not shown, as both techniques go to infinity when this situation is approached.)

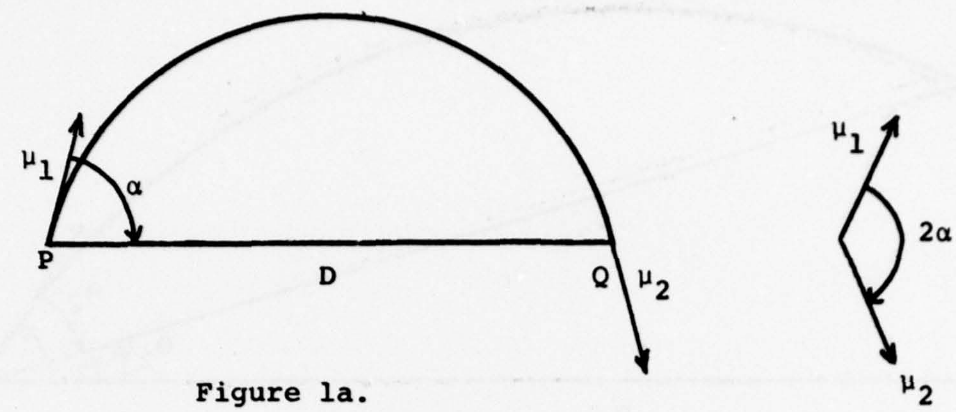


Figure 1a.

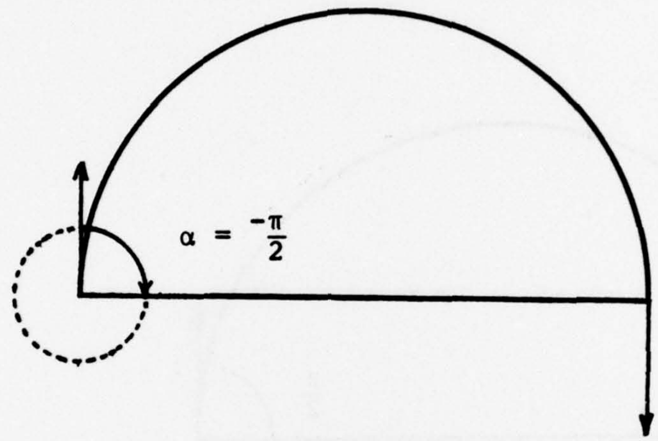


Figure 1b.

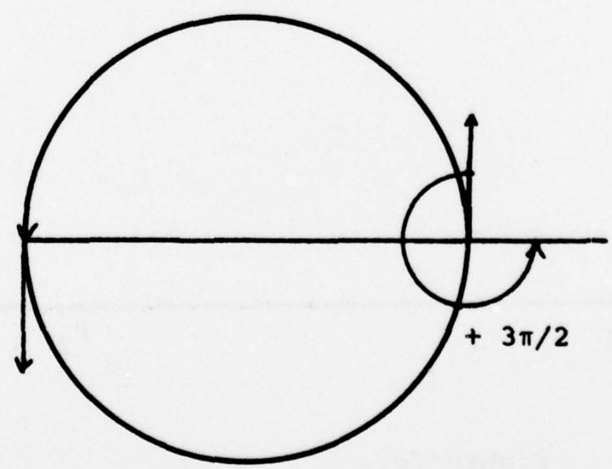


Figure 1c.

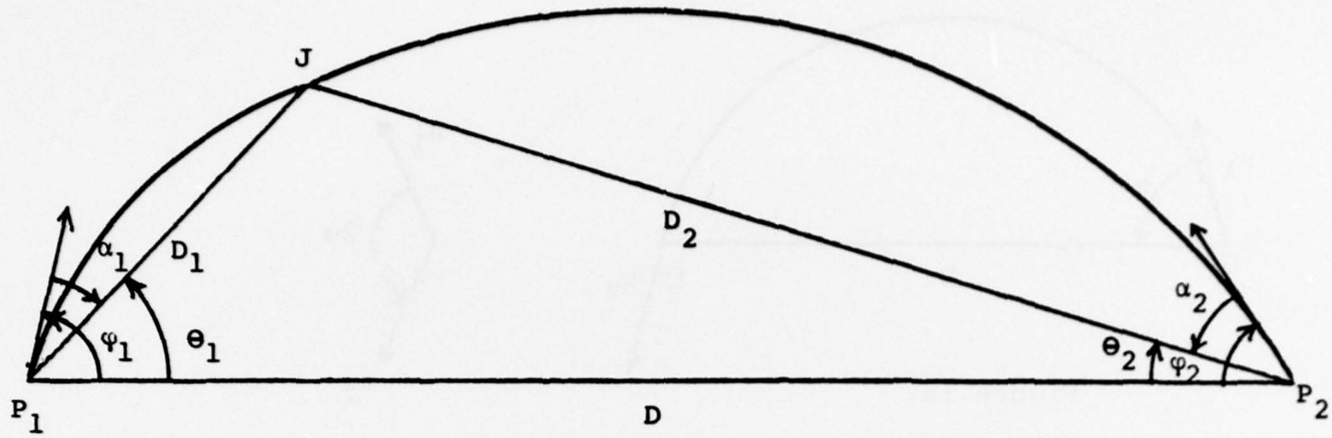


Figure 2a.

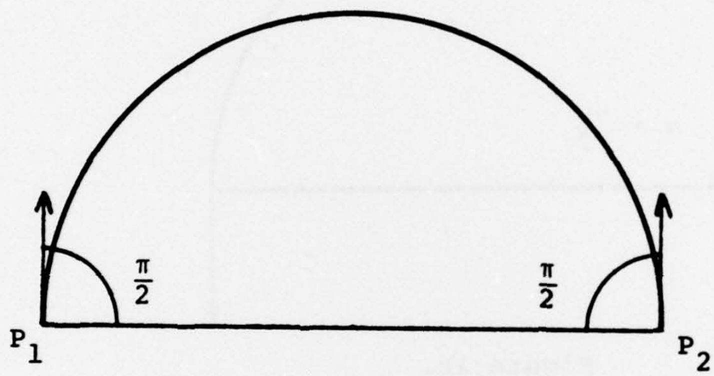
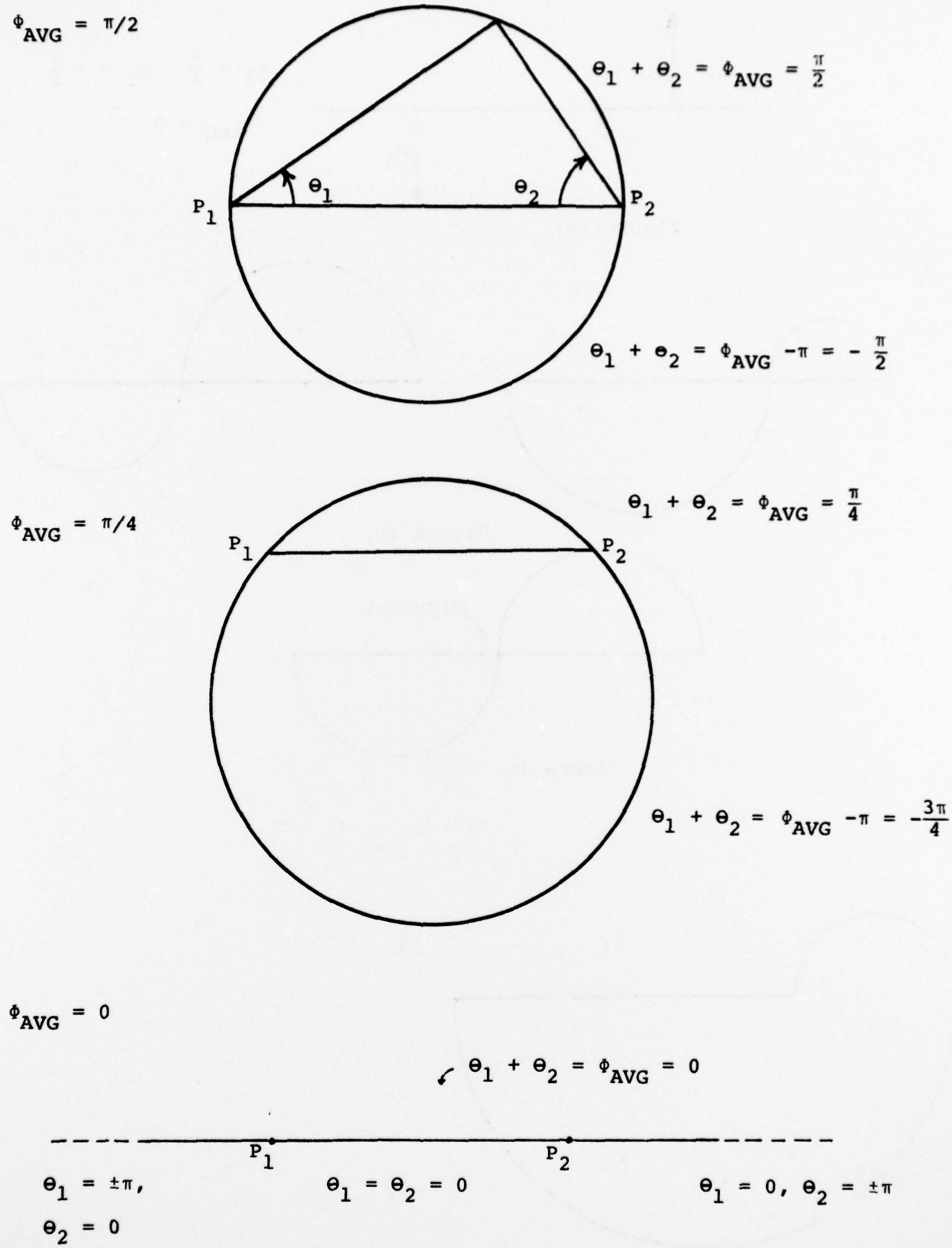


Figure 2b.

Figure 2c.



$\Phi_{AVG} = \pi/2$
$\Theta_1 + \Theta_2 = \Phi_{AVG} = \frac{\pi}{2}$
P_1
Θ_1
Θ_2
P_2
$\Theta_1 + \Theta_2 = \Phi_{AVG} - \pi = -\frac{\pi}{2}$
$\Theta_1 + \Theta_2 = \Phi_{AVG} = \frac{\pi}{4}$
$\Phi_{AVG} = \pi/4$
P_1
P_2
$\Theta_1 + \Theta_2 = \Phi_{AVG} - \pi = -\frac{3\pi}{4}$
$\Phi_{AVG} = 0$
$\Theta_1 + \Theta_2 = \Phi_{AVG} = 0$
P_1
P_2
$\Theta_1 = \pm\pi,$
$\Theta_2 = 0$
$\Theta_1 = \Theta_2 = 0$
$\Theta_1 = 0, \Theta_2 = \pm\pi$

Figure 3.

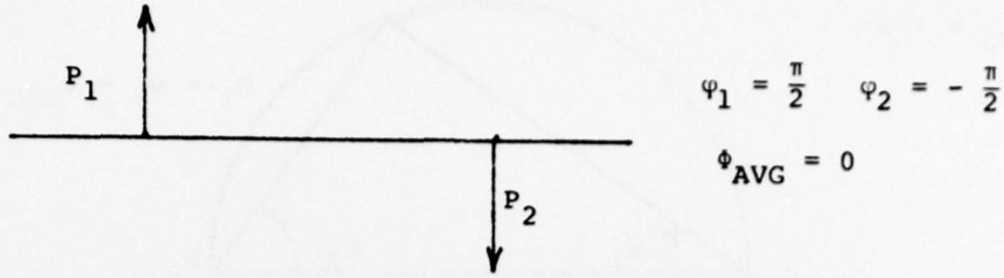


Figure 4a.

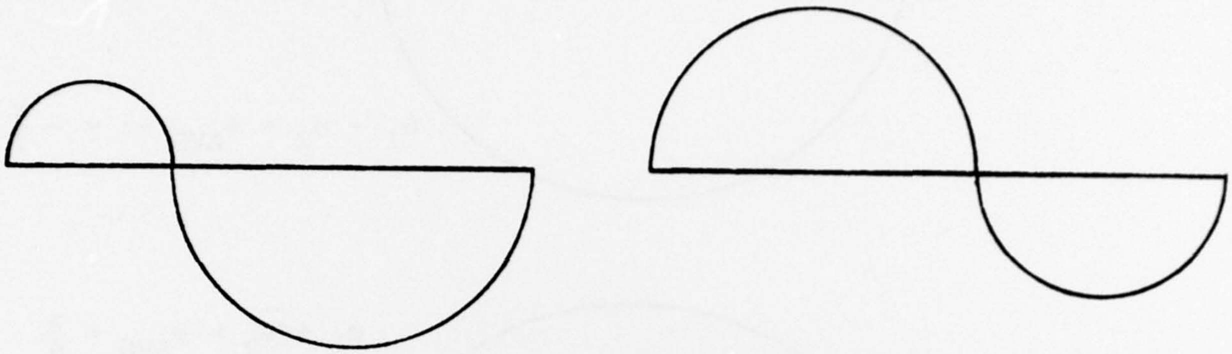


Figure 4b.

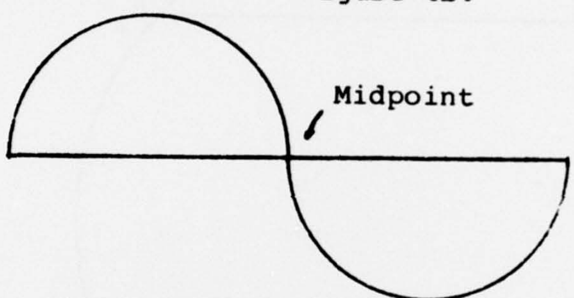


Figure 4c.

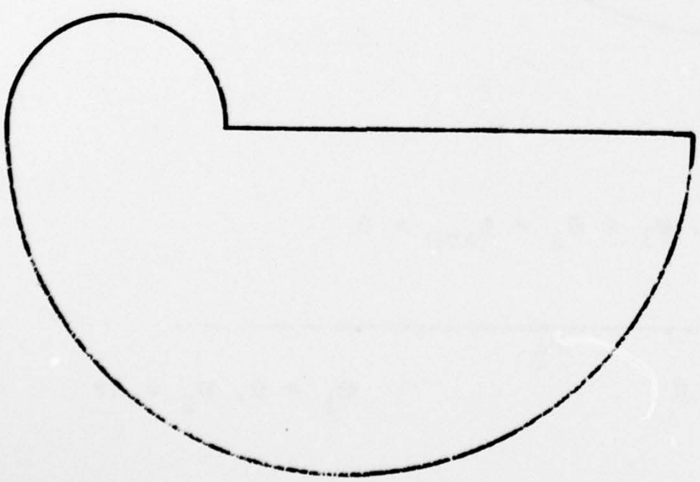


Figure 4d.

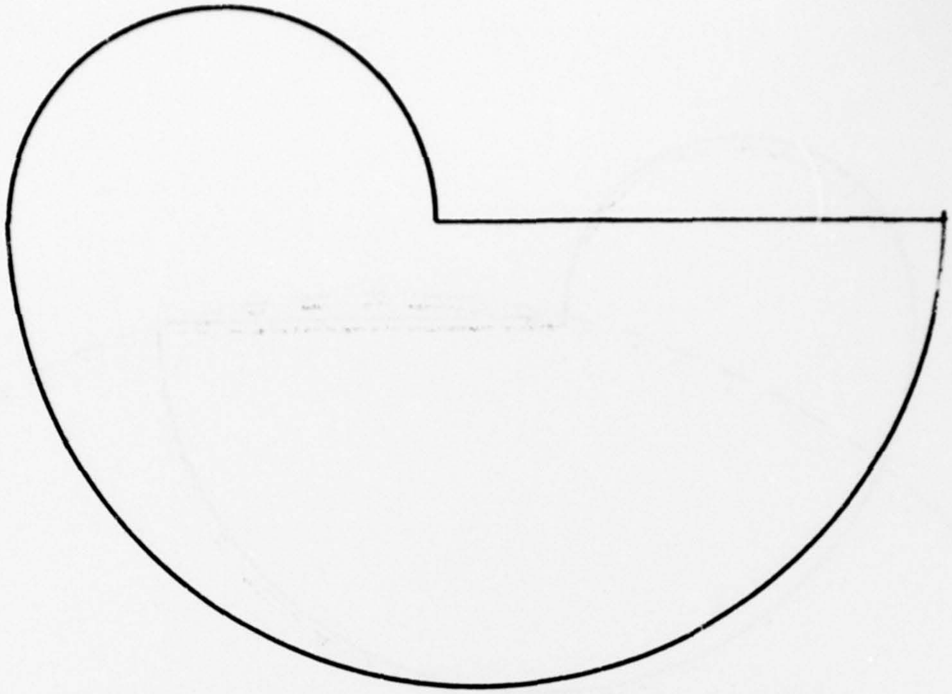
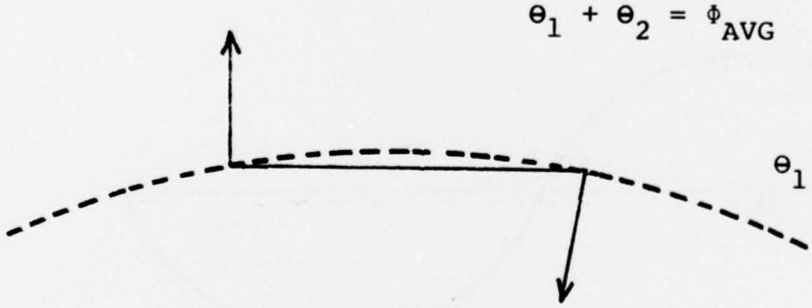


Figure 4d, cont'd.

$$\varphi_1 = \frac{\pi}{2}$$
$$\varphi_2 > -\frac{\pi}{2}$$

Figure 5a.

$$\Theta_1 + \Theta_2 = \Phi_{AVG}$$



$$\Theta_1 + \Theta_2 = \Phi_{AVG} - \pi$$

Figure 5b.

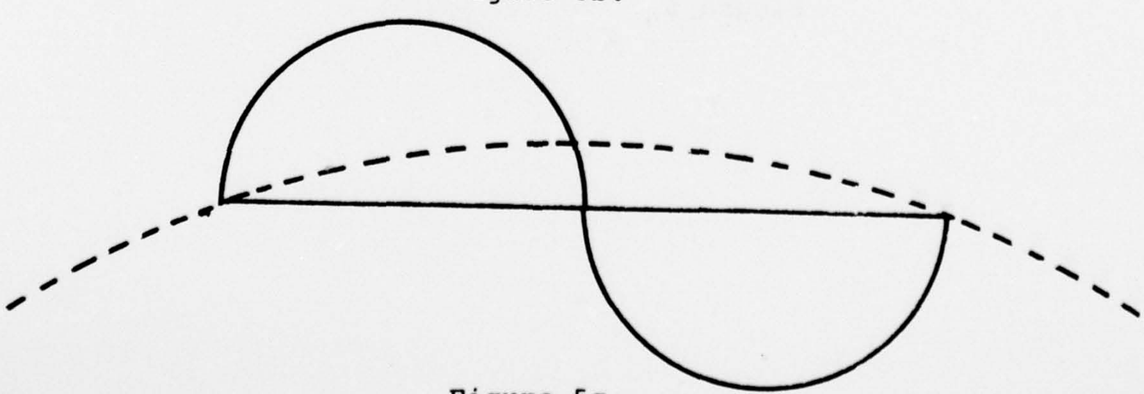


Figure 5c.

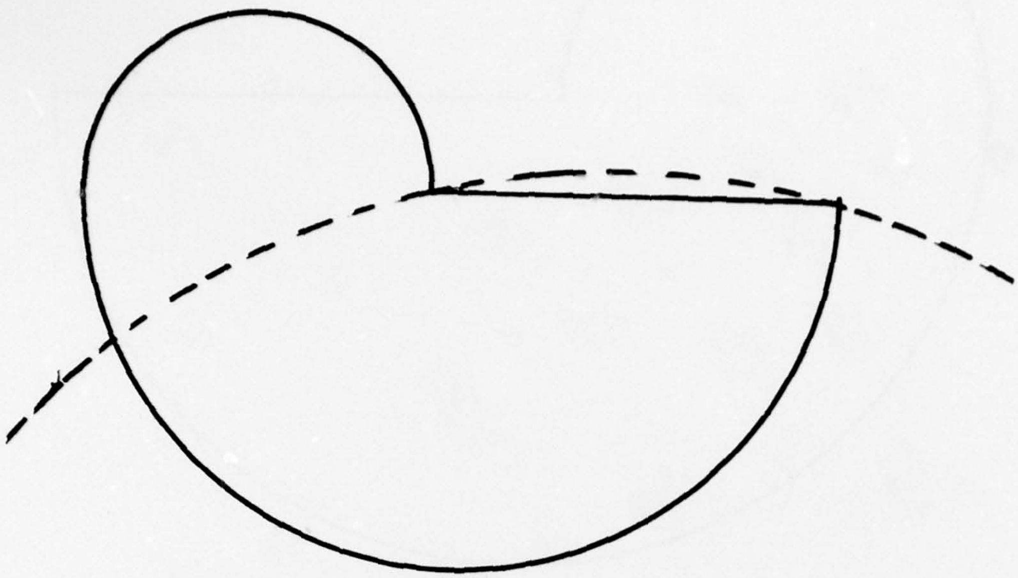


Figure 5d.

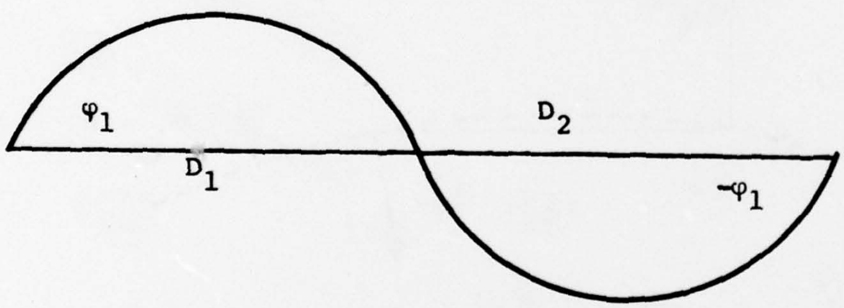
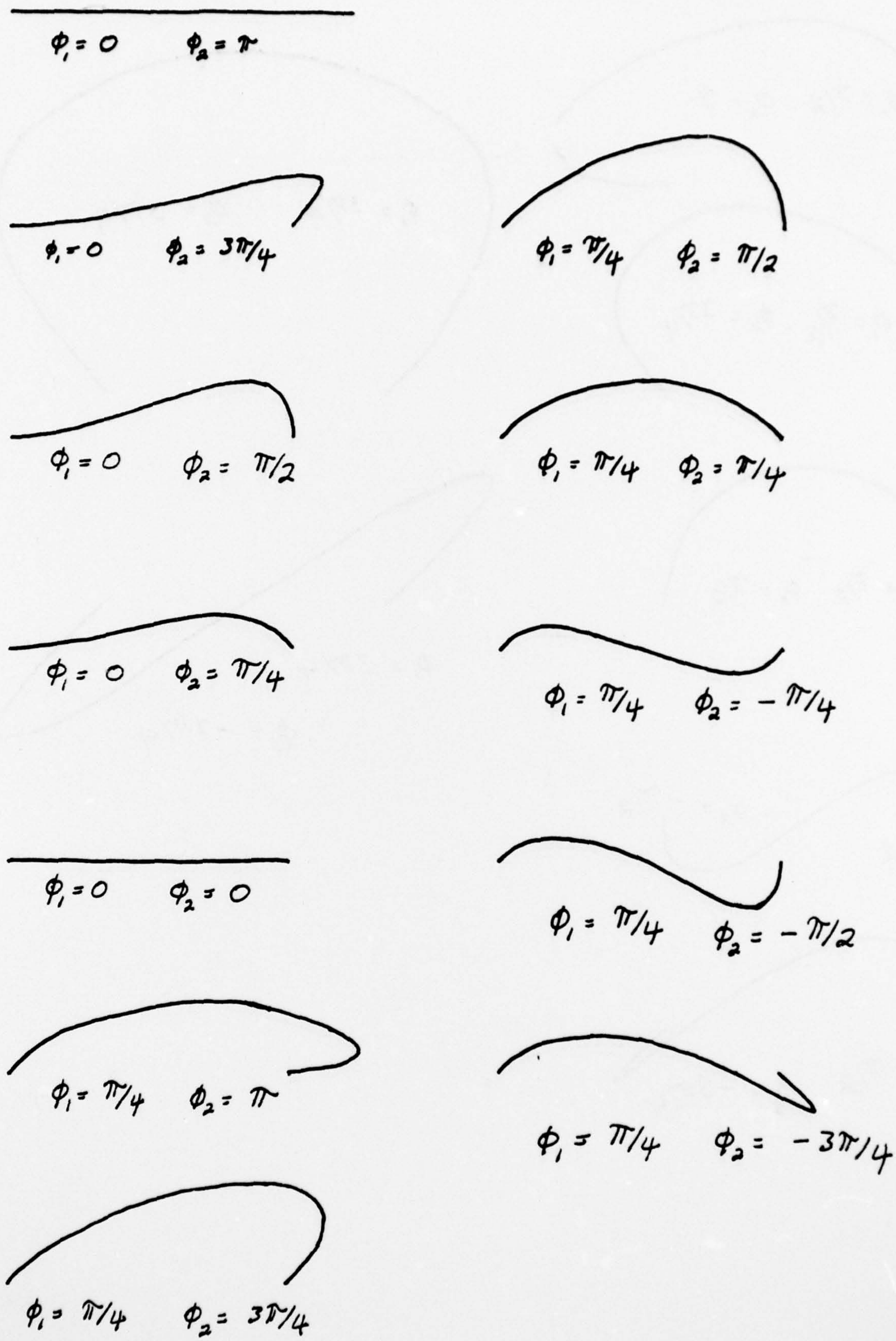
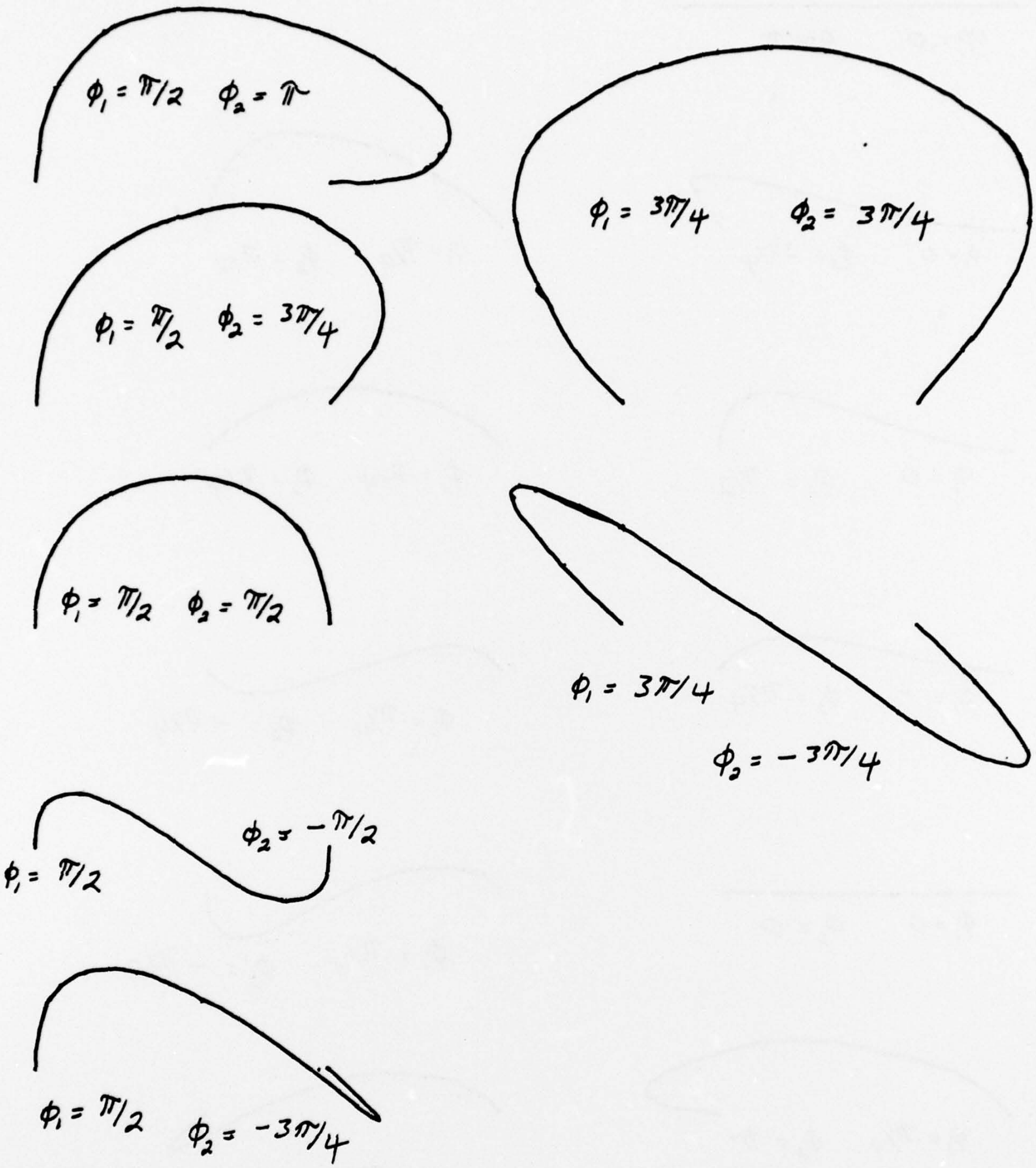


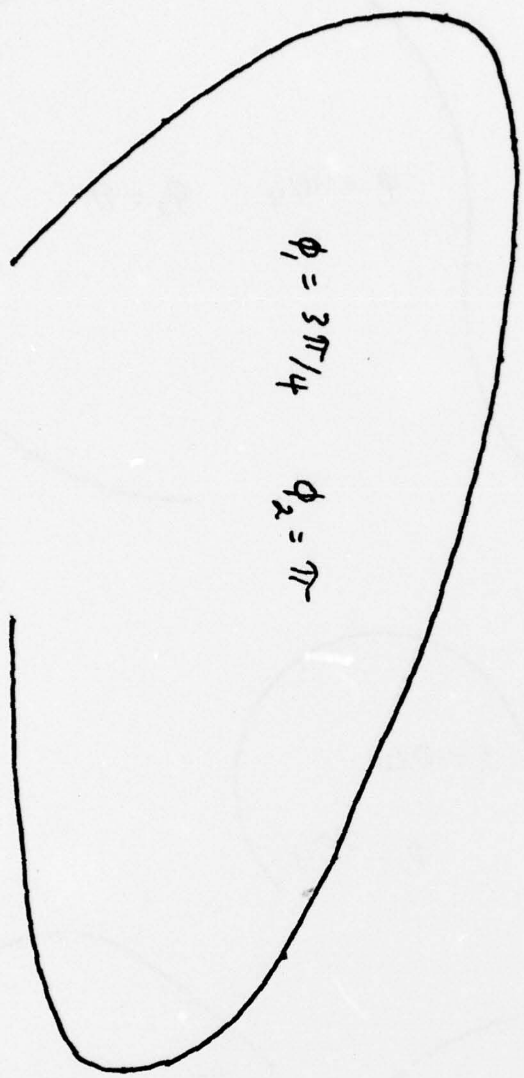
Figure 6.

Polynomial completions.

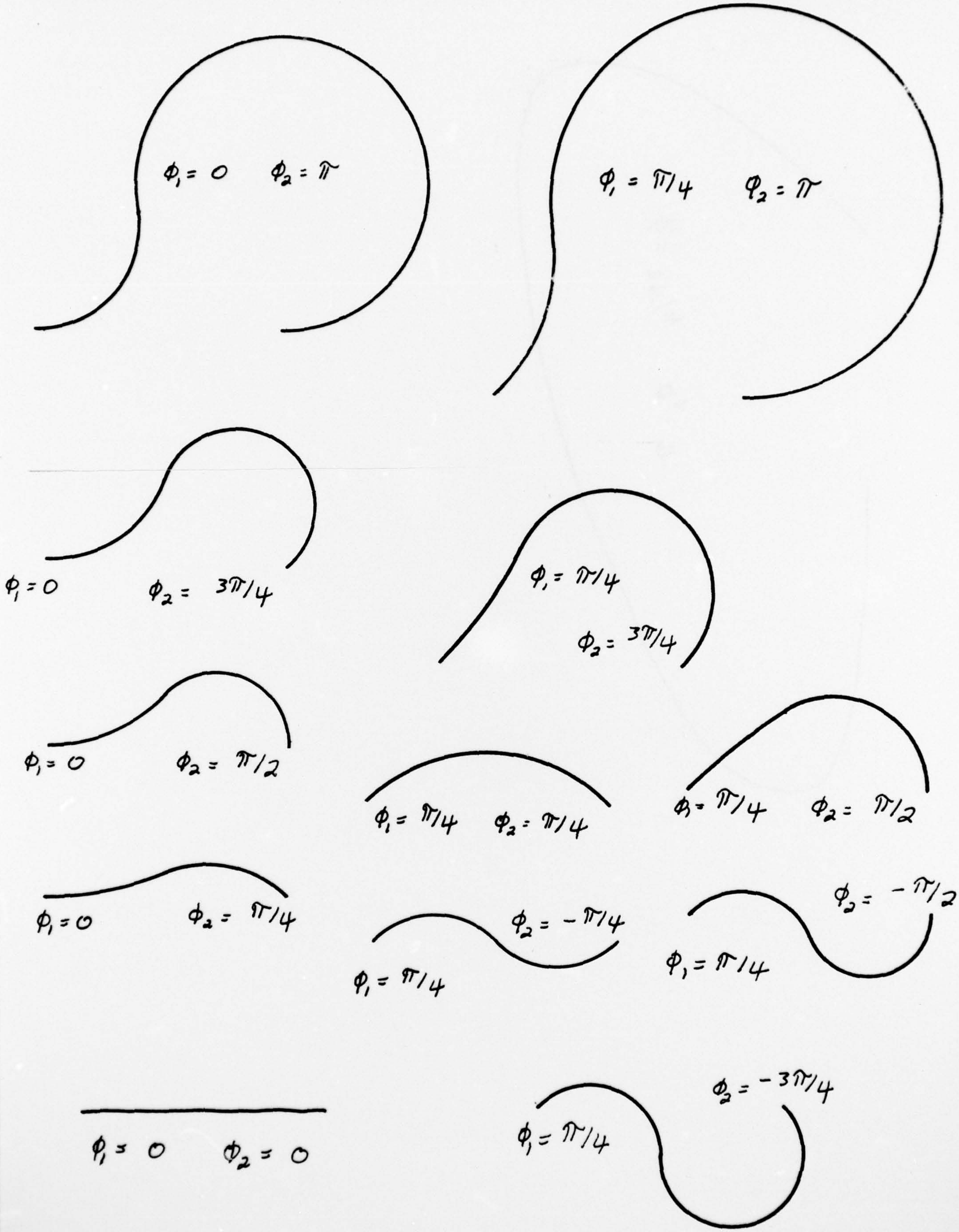


Polynomial completions (cont'd)

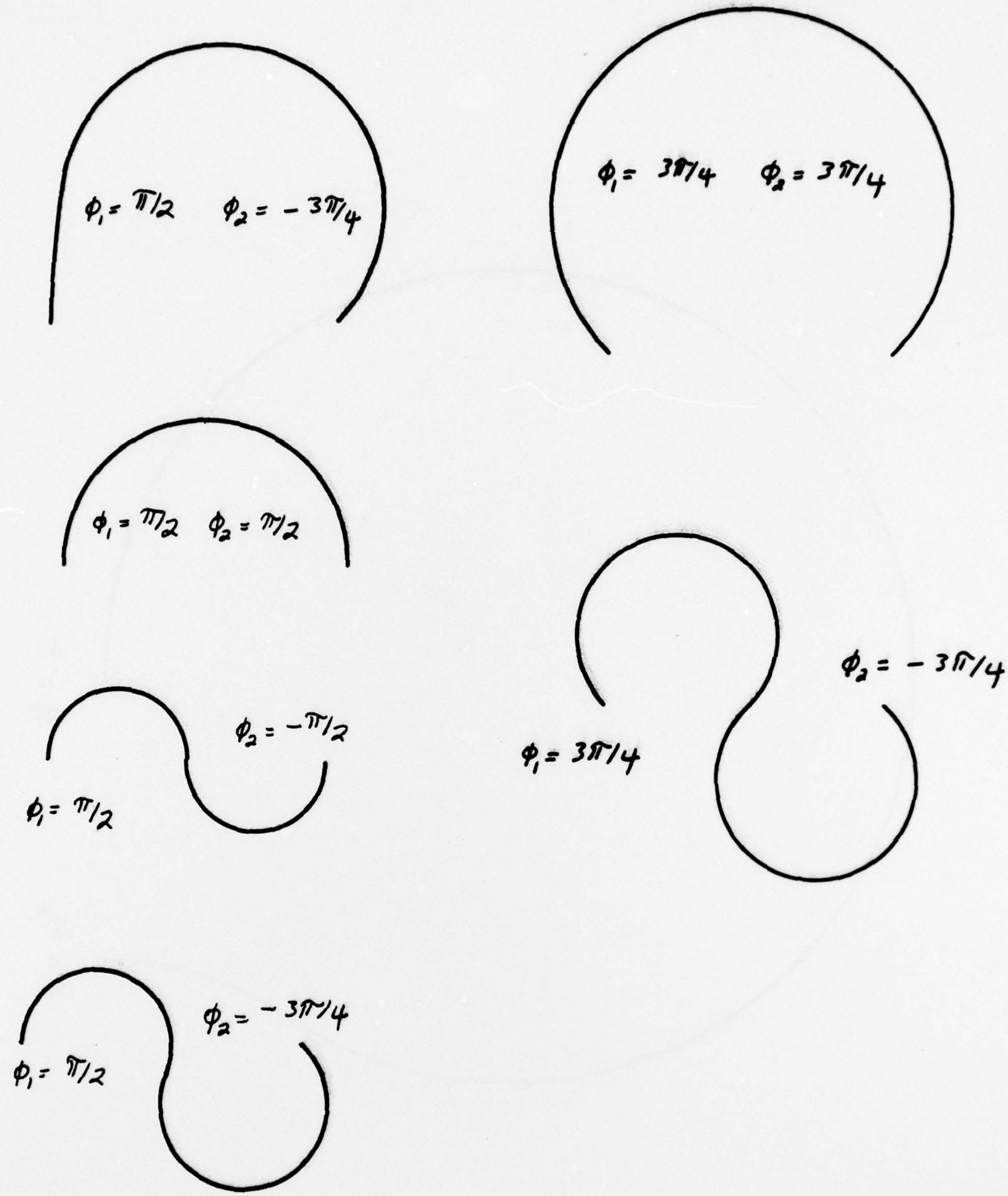



$\phi_1 = 3\pi/4$ $\phi_2 = \pi$

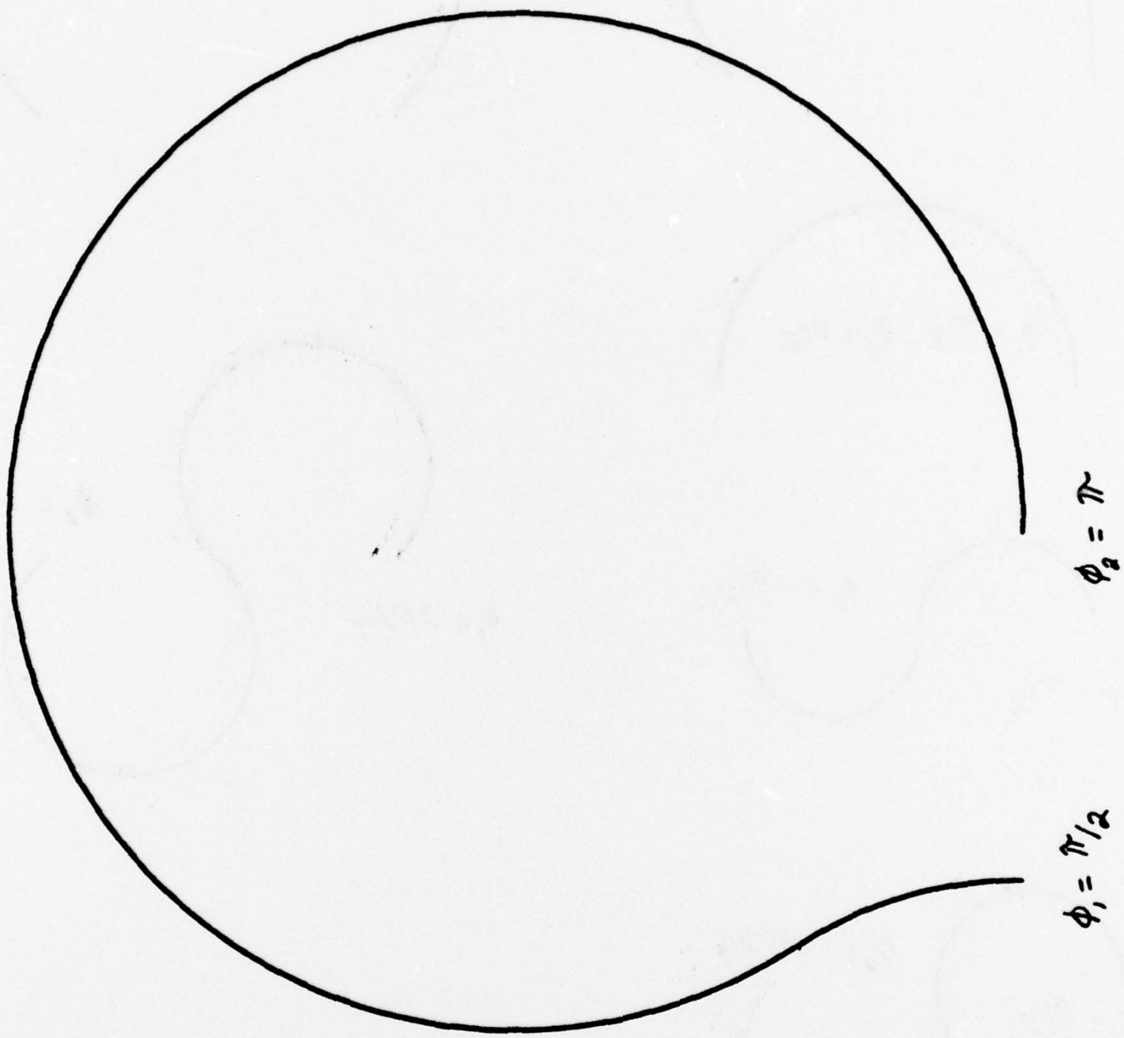
Circular completions.



Circular completions (cont'd)



Circular completions (cont'd)



Circular completions
(cont'd)

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) (6) SHAPE COMPLETION.		5. TYPE OF REPORT & PERIOD COVERED (9) Technical rept.,
		6. PERFORMING ORG. REPORT NUMBER (14) TR-564
7. AUTHOR(s) (10) Wallace/Rutkowski		8. CONTRACT OR GRANT NUMBER(s) (15) AFOSR-77-3271
9. PERFORMING ORGANIZATION NAME AND ADDRESS Computer Science Ctr. Univ. of Maryland College Park, MD 20742		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Math.& Info. Sciences, AFOSR/NM Bolling AFB Wash., DC 20332		12. REPORT DATE (11) Aug[redacted]77
		13. NUMBER OF PAGES (12) 35p.
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

403 018

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Pattern recognition
Image processing
Scene analysis
Shape description

Hul

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report describes some simple techniques for smoothly filling in gaps in object contours. The first technique considered was recently proposed by Ullman [1]; it constructs the completion of the contour using two arcs of circles that are tangent to the gap ends and to each other, and that have minimum total curvature. An analysis of this technique is presented, and examples of its use are given. A second technique uses cubic polynomial completions; when suitably constrained, this technique yields very reasonable completions.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)